coordinates. Substituting Eq. (14) into (5) and premultiplying the result by $\{\Psi^{(s)}\}^T$ gives

$$\sum_{r=1}^{2N} \{\Psi^{(s)}\}^T[M]\{\Phi^{(r)}\}\dot{\eta}_r(t) + \sum_{r=1}^{2N} \{\Psi^{(s)}\}^T[K]\{\Phi^{(r)}\}\eta_r(t) = \{\Psi^{(s)}\}^T\{Y(t)\}$$

which, in view of Eq. (13), reduces to the set of $2N$ uncoupled equations

$$\dot{\eta}_s(t) - \lambda_s\eta_s(t) = \frac{1}{M_s^*} F_s^*(t) \qquad (15)$$

where

$$F_s^*(t) = \{\Psi^{(s)}\}^T\{Y(t)\}$$

for $s = 1, \ldots, 2N$. The particular solution of Eq. (15) is given by the convolution integral

$$\eta_s(t) = \frac{1}{M_s^*} \int_0^t e^{\lambda_s\tau}F_s^*(t-\tau)d\tau \qquad (16)$$

and thus the set of normal coordinates are determined.

Finally, substituting Eq. (16) into (14) gives

$$\{y(t)\} = \sum_{r=1}^{2N} \frac{\{\Phi^{(r)}\}}{M_r^*} \int_0^t e^{\lambda_r\tau}F_r^*(t-\tau)d\tau$$

which, in terms of the elements of $\{y(t)\}$ and $\{\Phi^{(r)}\}$, is

$$\begin{bmatrix}\{\dot{q}(t)\}\\ \{q(t)\}\end{bmatrix} = \sum_{r=1}^{2N} \frac{1}{M_r^*} \begin{bmatrix}\lambda_r\{\phi^{(r)}\}\\ \{\phi^{(r)}\}\end{bmatrix} \int_0^t e^{\lambda_r\tau}F_r^*(t-\tau)d\tau \qquad (17)$$

Equation (17) is the result sought.

Conclusion

It is clear that the steady-state generalized nodal response is simply

$$\{q(t)\} = \sum_{r=1}^{2N} \frac{\{\phi^{(r)}\}}{M_r^*} \int_0^t e^{\lambda_r\tau}F_r^*(t-\tau)d\tau \qquad (18)$$

For stable systems, the eigenvalues are complex with a negative real part for underdamped modes. In this case, both the eigenvalues and eigenvectors occur in complex conjugate pairs and Eq. (18) can be expressed in real form by N terms.

In conclusion, this paper points out that the equations of motion of a flexible airplane can be written in the form of a general nonself-adjoint system which can be solved by the eigenfunction expansion.

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Errata

Drift of Buoyant Wing-Tip Vortices

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Equations (21, 23, and 24) should be corrected to read

$$\Gamma^2/a^3g > 4\pi^2[1 + (\lambda/2)] \qquad (21)$$

$$\frac{\Gamma^2}{a^3g} > \begin{cases} 4\pi^2[1 + (\lambda/2)] & (\lambda < 0.772) \\ 7.18\pi^2\lambda & (\lambda > 0.772) \end{cases} \qquad (23)$$

$$\langle\dot{X}\rangle_{\max} = \begin{cases} -(\lambda/2)\{ag/[1 + (\lambda/2)]\}^{1/2} & (\lambda < 0.772) \\ -(ag\lambda/7.18)^{1/2} & (\lambda > 0.772) \end{cases} \qquad (24)$$

These corrections do not alter any of the other results or conclusions.

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